Causal History Loss Upon Traversing the Original Einstein-Rosen Bridge: Horizon Effects and the Limits of Multi-Wormhole Recovery

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June 11, 2026

The original Einstein-Rosen bridge, proposed in 1935 to connect two asymptotically flat sheets of spacetime, has traditionally been viewed through the maximally extended Schwarzschild geometry, where the bridge is non-traversable and the exterior regions remain causally disconnected by horizons. Recent analysis reinterprets the 1935 solution as a lightlike thin-shell wormhole in which a localized exotic-matter shell at the throat renders the structure traversable while retaining the horizon structure. The corresponding Kruskal-Penrose diagram shows two exterior regions joined at an identified throat, with future and past horizons linked so that passage is possible. When an observer crosses the throat, the past horizon permanently excludes part of the global causal past from the traveler's new light cone; events whose null geodesics terminate behind this horizon become inaccessible, producing an irreversible loss of causal history even though the bridge itself is traversable. This raises the question whether a second wormhole can reconnect the observer with the lost portion of the past. Within the lightlike thin-shell framework the answer is negative unless the second bridge is specially engineered to generate closed timelike curves or to bypass the horizon filtering. Horizon identification, shell stability, and chronology-protection effects together constrain recovery, clarifying the causal cost of traversability in the original Einstein-Rosen construction and identifying the additional structures required if lost history is ever to be restored.

Keywords: Einstein-Rosen bridge, traversable wormhole, lightlike thin-shell wormhole, past horizon, causal history loss, closed timelike curves, chronology protection, multi-wormhole spacetime, horizon identification, thin-shell stability.

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Outline

1. The Original Einstein-Rosen Bridge and Its Modern Reinterpretation as a Lightlike Thin-Shell Wormhole

2. The Kruskal-Penrose Diagram for the Identified-Horizon Construction

3. Horizon Identification and the Mechanism of Traversability

4. Global Causal Structure of the Two-Sheeted Spacetime

5. Loss of Past Causal History When Crossing the Throat

6. The Filtering Role of the Past Horizon

7. Why a Second Identical Wormhole Cannot Restore Lost History

8. Geometric Requirements for Recovering Causal Past with Additional Wormholes

9. Closed Timelike Curves in Multi-Wormhole Configurations

10. Stability of Lightlike Thin Shells Under Multiple-Bridge Perturbations

11. Implications for Information Carried by Observers and Signals

12. Conclusions and Remaining Open Questions

13. References

Art

1. The Original Einstein-Rosen Bridge and Its Modern Reinterpretation as a Lightlike Thin-Shell Wormhole

The Einstein-Rosen bridge first appeared in 1935 as part of an attempt to eliminate singularities from the description of charged particles within a unified field theory. Einstein and Rosen considered a solution in which two copies of asymptotically flat spacetime are joined through a narrow throat, forming a bridge that replaces the singular point with a regular geometric connection between two sheets. In their construction the throat was static at the moment of time symmetry, and the geometry was intended to provide a nonsingular representation of what would otherwise appear as a point particle.

In the decades that followed, the same geometry was recovered as the maximal analytic extension of the Schwarzschild solution. In that standard reading the bridge connects two exterior regions that are separated by event horizons. The throat collapses on a timescale shorter than the light-crossing time, rendering the bridge non-traversable. Observers in one exterior region cannot send signals or travel to the other exterior region without encountering the future singularity. The two sheets therefore remain causally disconnected except at the formal level of the manifold topology.

A more recent line of analysis returns to the 1935 formulation and treats the bridge as a lightlike thin-shell wormhole. In this reinterpretation a thin shell of exotic matter, supported on a null hypersurface, is placed precisely at the throat. The shell supplies the negative energy density required to keep the throat open long enough for traversal. Because the shell is lightlike, it can be matched consistently to the vacuum regions on either side while preserving the horizon structure of the original solution. The resulting spacetime remains globally hyperbolic on each side of the shell, yet the identification of the future horizon on one sheet with the past horizon on the other sheet permits causal curves to cross from one exterior region into the other.

The thin-shell construction therefore restores traversability without removing the horizons. The Kruskal coordinates remain regular across the throat once the shell is included, and the conformal diagram displays two exterior regions joined at a single identified surface rather than separated by collapsing null boundaries. This configuration is the setting in which the loss of past causal history must be examined. An observer who crosses the throat does so through a region where the past horizon continues to act as a causal filter. Part of the global past that was visible before crossing lies behind this horizon relative to the new sheet and therefore drops out of the traveler’s future light cone. The loss is geometric and persists even though the bridge itself can be crossed.

The lightlike character of the shell is essential. A timelike or spacelike shell would alter the horizon structure or introduce additional instabilities. The null shell, by contrast, can be arranged so that the horizon identification survives while still allowing the throat to remain open for a finite proper time. This balance between traversability and the continued presence of horizons is what distinguishes the present framework from both the original 1935 static picture and the modern non-traversable Schwarzschild extension. It also sets the stage for asking whether additional wormholes can be introduced to recover the causal history that is lost upon a single crossing. The thin-shell model supplies a controlled arena in which that question can be posed without immediately violating energy conditions on each individual sheet.

Because the shell is localized and null, its stress-energy can be matched to the vacuum geometry using standard junction conditions. The resulting Israel surface stress tensor is supported only on the null surface and does not propagate into the exterior regions. Consequently the asymptotic flatness of both sheets is preserved, and the only modification to the causal structure occurs at the throat itself. This locality makes it possible to study the effect of horizon identification on past-directed geodesics in a clean manner. Null geodesics that reach the throat from one side can be continued into the other side, yet those that would have to originate from behind the past horizon on the original sheet cannot be recovered after the crossing. The geometric origin of the loss is therefore isolated to the horizon identification rather than to any global pathology of the manifold.

The reinterpretation also clarifies why the original Einstein-Rosen proposal, once equipped with the thin shell, behaves differently from both Morris-Thorne traversable wormholes without horizons and from the standard eternal black-hole wormhole. In the horizon-free case an observer can in principle maintain causal contact with the entire past of both mouths provided the mouths remain at rest relative to one another. In the present construction the retained horizons impose a permanent causal cut. The thin shell enables passage but does not erase the filtering action of the past horizon. This distinction is central to the subsequent analysis of whether a second wormhole can be arranged to reconnect the observer with the missing portion of the past. The single-bridge lightlike thin-shell model therefore serves as the minimal arena in which both traversability and irreversible loss of causal history coexist.

2. The Kruskal-Penrose Diagram for the Identified-Horizon Construction

The Kruskal-Penrose diagram provides the global causal structure of the lightlike thin-shell Einstein-Rosen bridge in a single conformal representation. Unlike the standard four-region diagram of the vacuum Schwarzschild extension, the present construction contains only two exterior regions joined through an identified throat. The diagram therefore consists of two

diamond-shaped blocks placed side by side and connected along their inner null boundaries. Each diamond corresponds to one asymptotically flat sheet. The right-hand diamond is conventionally labeled Region I with the time-like coordinate parameter satisfying eta greater than zero. The left-hand diamond is labeled Region II with the same parameter satisfying eta less than zero. The sign change across the central surface encodes the passage through the throat.

The coordinates used to draw the diagram are the Kruskal null coordinates. These are constructed so that the metric remains regular across the horizons. In standard notation the null coordinates are written as U and V, or equivalently w and v in some presentations. The radial coordinate r is expressed as a function of the product UV, while the time coordinate appears through their ratio. Because the coordinates are regular at the horizons, the diagram can display the entire maximal extension without coordinate singularities at the throat. The conformal factor that compactifies infinity is chosen so that null geodesics appear as straight lines at plus or minus forty-five degrees. This property makes causal relations immediately visible.

The two diamonds are bounded by four null lines that meet at the center. The upper pair of lines represents the future horizons, while the lower pair represents the past horizons. In the lightlike thin-shell construction the future horizon of Region I is identified with the past horizon of Region II, and vice versa. This identification is the geometric feature that permits causal curves to cross from one sheet to the other. The central intersection point of the four null lines corresponds to the location of the thin shell at the moment of time symmetry. An observer or a light ray that reaches this central surface from either side can be continued smoothly into the opposite region.

The conformal boundaries of each diamond are labeled in the usual way. The top vertex of each diamond is future timelike infinity, denoted i-plus. The bottom vertex is past timelike infinity, denoted i-minus. The left and right vertices of each diamond are spatial infinities, denoted i-zero. The upper slanted sides are future null infinity, denoted script-I-plus, and the lower slanted sides are past null infinity, denoted script-I-minus. These boundary points compactify the asymptotic regions so that the global causal structure fits inside a finite figure while preserving the angles between null geodesics.

Because the diagram is drawn in Kruskal coordinates, the radial function r can be read off along any spacelike slice. The throat itself appears as the minimal surface where r equals twice the mass parameter. On a surface of constant Kruskal time the throat is a closed two-sphere of this minimal radius. The identification across the throat means that an observer who crosses this minimal surface enters the second sheet without encountering a singularity. However, the continued presence of the past horizon implies that not every past-directed null geodesic from the original sheet reaches the new sheet. Geodesics that would have to originate from behind

the past horizon on the departure side terminate at the horizon and therefore do not connect through the throat.

The filtering action of the past horizon becomes geometrically transparent in the diagram. Consider a null geodesic that leaves an event in Region I and travels toward the throat. If the geodesic lies to the future of the past horizon in that region, it can cross the identified surface and continue into Region II. If the geodesic lies to the past of the same horizon, it cannot reach the throat at all; it either falls into the past singularity or remains confined to Region I. Consequently, an observer who has already crossed into Region II loses causal contact with all events whose past light cones are cut off by the past horizon of the original sheet. This loss is permanent and is visible directly on the diagram as the region behind the past horizon that has no connecting null lines to the opposite diamond.

The diagram also makes clear why the construction differs from both the static 1935 picture and the modern non-traversable extension. In the static picture the throat is held open without horizons, but the solution is not asymptotically flat on both sides in the required way. In the vacuum Schwarzschild extension the horizons are not identified for traversal; they separate two disconnected exteriors. In the lightlike thin-shell version the horizons are retained yet identified, producing a hybrid structure in which passage is possible while the causal filtering of the past horizon survives. The diagram therefore encodes both the traversability and the irreversible loss of past causal history in a single compact figure.

Because the coordinates are regular across the throat, one can draw families of geodesics that cross from one diamond into the other. Timelike worldlines that reach the central surface with future-directed tangent vectors continue into the opposite region. Their future light cones after crossing lie entirely within the new diamond and do not extend back into the portion of the original diamond that lies behind the past horizon. This geometric fact is the diagrammatic representation of the statement that part of the traveler's causal past is lost upon traversal. The loss is not an artifact of coordinate choice; it follows directly from the horizon identification that is required for the thin shell to match consistently to the vacuum regions on both sides.

The same diagram can be used to examine the effect of adding a second wormhole. Any additional bridge must be drawn as another pair of identified null surfaces connecting the two diamonds or connecting one diamond to itself in a more complicated topology. Because each such surface must respect the same horizon structure, the second bridge cannot reach the region behind the past horizon of the first crossing without violating the causal properties already fixed by the diagram. The figure therefore supplies a visual criterion for whether any proposed multi-wormhole configuration can restore the missing history: the new null surfaces must connect points that lie to the future of the original past horizon in a manner that creates

closed timelike curves. The diagram makes the topological and causal obstacles to such a construction immediately apparent.

In summary, the Kruskal-Penrose diagram for the identified-horizon lightlike thin-shell bridge displays the two exterior regions, the regular Kruskal coordinates, the identified future and past horizons, and the central throat in a single conformal picture. It renders the traversability of the bridge visible while simultaneously showing that the past horizon continues to filter causal past upon crossing. This combination of features distinguishes the construction from both non-traversable and horizon-free wormholes and provides the geometric foundation for analyzing whether additional wormholes can recover the lost portion of an observer's causal history.

3. Horizon Identification and the Mechanism of Traversability

In the lightlike thin-shell construction of the original Einstein-Rosen bridge, traversability is achieved through a specific identification of the horizons rather than through the removal of horizons. The future horizon on one sheet is matched to the past horizon on the other sheet across the null shell that occupies the throat. This identification allows causal curves to continue from one exterior region into the other without encountering a singularity or a collapsing throat. The mechanism is local to the shell and does not alter the asymptotic flatness of either sheet.

The identification procedure can be described in Kruskal coordinates. Let the two exterior regions be labeled Region I and Region II, with the Kruskal null coordinates (U, V) defined so that the product UV determines the radial function r. On the future horizon of Region I the coordinate V vanishes while U remains finite. On the past horizon of Region II the coordinate U vanishes while V remains finite. The thin shell is placed precisely on the surface where these two horizons coincide after the identification U of one side is set equal to V of the other side (up to a constant rescaling that preserves the null character). Once this matching is imposed, a future-directed null geodesic that reaches the shell from Region I continues as a future-directed null geodesic into Region II, and likewise for timelike curves that cross with sufficient velocity.

The lightlike character of the shell is essential to the consistency of this matching. Because the shell is null, its surface stress-energy tensor satisfies the junction conditions without introducing additional discontinuities in the extrinsic curvature that would destabilize the horizons. The Israel formalism applied to a null hypersurface yields a surface stress tensor whose only non-zero component lies along the null direction. This component can be chosen to be negative, supplying the exotic matter required to keep the throat from collapsing while leaving the vacuum geometry unchanged on each side of the shell. The resulting spacetime therefore

remains a solution of the Einstein equations everywhere except on the shell itself, where the thin-shell source is localized.

In the standard vacuum Schwarzschild extension the same horizons exist, but they are not identified for traversal. A future-directed curve that crosses the future horizon of one exterior region enters the black-hole interior and terminates at the future singularity. There is no continuation into the second exterior region. The throat pinches off before any signal can traverse it. In the thin-shell construction the identification removes this obstruction. The interior regions that would appear in the four-region diagram are effectively replaced by the direct matching across the shell. Consequently the global manifold contains only the two exterior regions connected at the throat, and the causal structure permits passage while the horizons themselves remain in place.

The continued presence of the horizons, even after identification, is what produces the irreversible loss of past causal history. Consider an observer who begins in Region I and crosses the shell into Region II. Before crossing, the observer's past light cone includes events in Region I whose null geodesics reach the throat from the past side of the past horizon. After crossing, the observer's new past light cone lies entirely within Region II. Any event in Region I whose connecting null geodesic would have to originate behind the past horizon of Region I is no longer reachable. The identification that allows the observer to enter Region II simultaneously prevents those particular past signals from following the observer across the throat. The loss is therefore a direct geometric consequence of the horizon identification rather than an artifact of the shell's energy condition.

The same identification governs the behavior of closed curves that might be constructed with additional wormholes. Any second bridge must also be realized as a null shell with its own horizon identification. Because each such identification respects the same causal boundary conditions, a second bridge cannot connect a point after the first crossing back to an event that lies behind the original past horizon without creating a closed timelike curve that violates the global hyperbolicity of each individual sheet. The horizon identification therefore sets a strict limit on the type of multi-wormhole configurations that can be consistently added while preserving the traversability of each individual bridge.

It is useful to contrast this mechanism with horizon-free traversable wormholes. In those constructions the absence of horizons allows an observer to maintain causal contact with the entire past of both mouths provided the mouths remain at fixed separation. The lightlike thin-shell Einstein-Rosen bridge retains horizons precisely because it descends from the 1935 solution that already contained them. The shell supplies the minimal exotic matter needed for traversability without eliminating the horizons. The resulting structure is therefore intermediate: it permits passage yet enforces a permanent causal cut on the past. This

intermediate character is what makes the question of recovering lost history with a second wormhole nontrivial. The second wormhole must overcome not only the energy-condition requirements but also the causal filtering already imposed by the identified past horizon of the first bridge.

The matching across the shell can be made explicit in terms of the Kruskal coordinates. Let the future horizon of Region I be the surface V = 0 with U greater than zero. After identification this surface is glued to the past horizon of Region II, which is the surface U = 0 with V greater than zero. The gluing map sends points with the same value of the remaining coordinate across the shell. A future-directed vector that is tangent to a geodesic on one side continues smoothly into the other side because the metric coefficients remain finite and the identification preserves the null character of the horizons. The only additional ingredient is the surface stress tensor on the shell, whose magnitude is fixed by the requirement that the throat remain open for the duration of the crossing. Once this magnitude is chosen, the entire causal structure, including the filtering action of the past horizon, follows uniquely from the geometry.

In this way the horizon identification supplies both the traversability and the mechanism of history loss. The same geometric feature that allows an observer to move from one sheet to the other guarantees that part of the observer's original past light cone cannot follow. Any attempt to restore the missing history with additional wormholes must therefore either bypass this identification or introduce closed timelike curves that connect the post-crossing region back to the pre-crossing past in a controlled manner. The lightlike thin-shell construction makes both the possibility of traversal and the price paid in lost causal history explicit and localizable to the throat.

4. Global Causal Structure of the Two-Sheeted Spacetime

The global causal structure of the lightlike thin-shell Einstein-Rosen bridge consists of two asymptotically flat exterior regions joined at a single traversable throat while remaining partially disconnected by the identified horizons. In this two-sheeted spacetime an observer in one region can send signals and travel into the other region, yet cannot maintain causal contact with the entire past of the original region after crossing. The structure therefore combines limited connectivity with a permanent causal filter.

Each exterior region possesses its own complete set of light cones. In Region I the future light cones point toward the future horizon and the identified throat. In Region II the future light cones point away from the throat into the new sheet. Because the horizons are identified across the null shell, a future-directed causal curve that reaches the throat from Region I continues into Region II as a future-directed curve. The same holds for past-directed curves that reach the

throat from Region II. This identification creates a one-way causal bridge between the two sheets. However, the identification does not extend to the entire past light cones of both regions. Events whose connecting null geodesics lie behind the past horizon on the departure side have no continuation across the throat.

The causal connectivity across the throat can be characterized by the behavior of null geodesics. A future-directed null geodesic originating in Region I and aimed at the throat will cross into Region II provided it lies to the future of the past horizon in Region I. Once across, the geodesic continues in Region II and can reach future null infinity there. Conversely, a past-directed null geodesic originating in Region II and aimed backward toward the throat will enter Region I only if it lies to the future of the past horizon in Region II after the identification. Geodesics that fail these conditions terminate at the past horizon and never connect the two sheets. Consequently the throat functions as a selective causal portal rather than a complete bridge between the full causal pasts of both regions.

The independence of the light cones on each sheet becomes most apparent when considering an observer who has already crossed. Before crossing, the observer's past light cone includes events in the original region that lie arbitrarily far in the past, provided those events can send signals that reach the throat. After crossing, the observer's new past light cone is confined to the destination region. Any event in the original region whose signal would have to cross the past horizon from the "wrong" side is excluded. This exclusion is not a dynamical process that occurs during the crossing; it is a fixed feature of the global causal structure. The diagram shows entire wedges of the original diamond that have no connecting null lines to the opposite diamond after the identification. Those wedges represent the lost causal history.

The two-sheeted spacetime therefore differs from both a fully disconnected pair of universes and a single connected spacetime without horizons. In a fully disconnected configuration no causal curves exist between the sheets at all. In a horizon-free traversable wormhole an observer can in principle remain in causal contact with events on both sides before and after crossing, provided the mouths are not moving relative to one another. In the present construction the retained and identified horizons create an intermediate situation: passage is possible, yet the causal past is permanently truncated upon traversal. The truncation is encoded in the horizon identification itself and does not require additional dynamical collapse of the throat.

Signals sent from one sheet to the other after a crossing also respect the same filtering. A light signal emitted in Region II after an observer has arrived there can reach future null infinity in Region II and can be received by other observers who remain in Region II. It cannot, however, propagate backward across the throat into the lost portion of Region I's past. Any attempt to send a signal that would reach an event behind the original past horizon would require the

signal to travel into the past relative to the crossing event, which is forbidden by the causal structure. The throat therefore transmits information forward in time from one sheet to the other but does not allow recovery of information that was already behind the past horizon at the moment of crossing.

The global causal structure also constrains the possibility of using additional wormholes to restore lost history. Any second bridge must be realized as another null shell with its own horizon identification. Because each identification preserves the same causal boundaries, a second throat can connect points that lie to the future of the original past horizon, but it cannot reach points that lie behind that horizon without creating a closed timelike curve that loops around the filtering surface. Such a loop would require the second bridge to connect a post-crossing event back to a pre-crossing event in a manner that violates the global hyperbolicity on at least one of the sheets. The causal structure therefore imposes a topological obstruction in addition to the energy-condition requirements already present for each individual shell.

In summary, the two-sheeted spacetime defined by the lightlike thin-shell Einstein-Rosen bridge possesses a causal structure in which the two exterior regions are connected by an identified throat that permits traversal while the past horizon continues to filter causal past. Observers who cross lose permanent access to the portion of their original past that lies behind the past horizon. Signals and worldlines can move from one sheet to the other in the forward time direction, but they cannot recover information or events that were already causally inaccessible at the moment of crossing. This structure is the geometric setting in which the question of regaining lost history with additional wormholes must be posed, and it shows why such recovery requires configurations that go beyond simple duplication of the original bridge.

5. Loss of Past Causal History When Crossing the Throat

When an observer crosses the throat of the lightlike thin-shell Einstein-Rosen bridge, part of the global causal past becomes permanently inaccessible. This loss is not caused by any dynamical instability of the throat or by the energy condition of the shell. It follows directly from the presence and identification of the past horizon. The observer who begins in one exterior region and enters the second region finds that certain events which were visible before the crossing no longer lie within the new past light cone.

Consider an observer at rest in Region I at a finite distance from the throat. Before crossing, the observer’s past light cone extends backward in time and includes events arbitrarily far in the past within Region I, provided those events can emit null geodesics that reach the observer’s worldline. In the Kruskal-Penrose diagram these events correspond to points in the right-hand

diamond that lie to the future of the past horizon. Their connecting null geodesics can be drawn as straight lines at forty-five degrees that reach the central throat surface.

At the moment the observer crosses the identified throat and enters Region II, the causal structure changes abruptly from the observer's perspective. The new past light cone is now constructed entirely within Region II. Any null geodesic that would have to originate from a point in Region I lying behind the past horizon cannot continue across the throat. In the diagram these geodesics terminate at the past horizon line and have no continuation into the left-hand diamond. Consequently the observer loses causal contact with all events whose past-directed null geodesics are cut off by that horizon.

The loss is irreversible because the horizon identification that permits traversal simultaneously defines a causal boundary that cannot be crossed in the reverse direction for past-directed signals. Once the observer has moved into Region II, there is no future-directed path within the new region that can reach back behind the original past horizon of Region I. The throat allows forward passage but does not provide a return route to the filtered portion of the past. This one-way character is encoded in the relative orientation of the identified horizons: the future horizon on the departure side becomes the entry surface, while the past horizon on that same side remains a barrier for signals originating from earlier times.

The portion of the past that is lost can be characterized geometrically. In Kruskal coordinates the past horizon corresponds to the surface on which one of the null coordinates vanishes. Events with Kruskal time coordinate values sufficiently negative relative to the crossing event lie on the wrong side of this surface. Their future light cones do not intersect the throat after the identification has been imposed. An observer who has crossed therefore cannot receive light or any causal signal from those events, no matter how long they wait in the new region. The loss is not a matter of waiting long enough or traveling far enough; it is a fixed feature of the global causal structure.

This truncation of the past light cone distinguishes the lightlike thin-shell Einstein-Rosen bridge from both non-traversable and horizon-free wormholes. In the standard vacuum Schwarzschild extension the bridge collapses and no observer crosses at all. In horizon-free traversable wormholes an observer can maintain continuous causal contact with events on both sides before and after crossing. In the present construction the retained past horizon creates a sharp division: the observer carries forward only the causal past that lies to the future of the horizon at the moment of crossing. Everything else is left behind permanently.

The same loss occurs for any causal curve, not only for the observer's own worldline. A spaceship or a light signal that crosses the throat at a given event carries with it only the information that was contained in its past light cone up to that event. After crossing, no

additional information from the filtered region can catch up. If two observers cross at different times or along different trajectories, each loses a slightly different portion of the past depending on the precise location of their crossing relative to the past horizon. The loss is therefore observer-dependent in detail while remaining universal in character: every crossing observer loses some part of the original region's causal past.

Because the loss is caused by the horizon rather than by the thin shell itself, it persists even if the shell is made arbitrarily thin or its energy density is adjusted within the range allowed by the junction conditions. The shell enables the throat to remain open and the identification to be consistent, but it does not alter the causal boundary defined by the past horizon. Any attempt to recover the lost history must therefore address the horizon structure directly rather than the properties of the shell alone. This fact becomes important when considering whether a second wormhole can be introduced to reconnect the traveler with the missing portion of the past. The second bridge would have to overcome the same causal filtering that the first bridge already imposes.

In the Kruskal-Penrose diagram the lost history appears as the wedge of Region I that lies below the past horizon and has no connecting null lines to Region II after the identification. This wedge is causally disconnected from all future events in Region II. An observer who has crossed into Region II can never receive signals from inside that wedge, regardless of the path taken or the time elapsed. The diagram therefore makes the loss visually immediate and shows why it is a structural feature of the spacetime rather than a contingent effect that could be avoided by different choices of trajectory or timing.

6. The Filtering Role of the Past Horizon

The past horizon functions as a one-way causal filter that permanently separates part of the global past from any observer who has crossed the throat. In the Kruskal-Penrose diagram it appears as the lower diagonal boundary of each diamond. Its orientation and identification with the corresponding horizon on the opposite sheet determine which past-directed null geodesics can reach the throat and which cannot. The filtering is not a dynamical process that develops during the crossing; it is a fixed property of the causal structure that becomes relevant the moment an observer moves from one sheet to the other.

Geometrically, the past horizon is the surface on which one of the Kruskal null coordinates vanishes while the other remains positive. In Region I this surface marks the boundary beyond which past-directed null geodesics cannot reach the throat after the identification has been imposed. Any event lying to the past of this surface has a future light cone that either terminates at the past singularity or remains confined within Region I. When the future horizon

of Region I is identified with the past horizon of Region II, the continuation of geodesics across the throat is allowed only for those that approach the surface from the allowed side. Geodesics originating from the filtered side have no continuation.

For an observer who crosses the throat, the filtering manifests as an abrupt truncation of the past light cone. Before crossing, the observer can in principle receive signals from events whose null geodesics reach the throat from the future side of the past horizon. After crossing, the new past light cone lies entirely in the destination region. The observer cannot send a signal backward across the throat to query events behind the original past horizon, nor can any signal from those events catch up after the crossing has occurred. The past horizon therefore acts as an event horizon for the observer's new causal past, even though the observer has successfully traversed into the second sheet.

The filtering is selective rather than total. Not all of the original region's past is lost. Events sufficiently close in time to the crossing event, or whose signals reach the throat well before the observer crosses, remain accessible in principle if their geodesics lie on the allowed side of the past horizon. The lost portion consists of events whose connecting null geodesics would have to cross the past horizon from the wrong direction. In the diagram these events occupy the wedge below the past horizon line in the departure diamond. That wedge has no null connections to the opposite diamond after identification.

The future horizon plays a complementary but distinct role. It limits what an observer in one region can send into the future of the other region after crossing. Because the future horizon is also identified, signals emitted after crossing can propagate forward into the new region without obstruction. The asymmetry between the past and future horizons is what produces the one-way character of the history loss: the past is filtered at the moment of crossing, while the future remains open in the destination region.

This filtering persists for all causal curves, not only for the worldlines of massive observers. A light signal or a spaceship that crosses the throat carries only the causal information that was contained in its past light cone up to the crossing event. Any information originating behind the past horizon on the departure side cannot be carried across. If two observers cross at different Kruskal times, each loses a different subset of the past depending on the precise location of their crossing relative to the horizon. The filtering is therefore both universal in its geometric origin and individual in its detailed consequences for each traveler.

The continued action of the past horizon after identification distinguishes the lightlike thin-shell construction from horizon-free traversable wormholes. In the latter, an observer can maintain continuous causal contact with events on both sides before and after crossing. In the present model the retained and identified past horizon creates a sharp causal cut. The throat permits

spatial passage between the two sheets, but it does not permit the full causal past to follow. This cut is what makes the loss of history irreversible within the single-bridge spacetime.

Because the filter is defined by the horizon itself rather than by the thin shell, it cannot be removed by adjusting the shell's energy density or thickness within the allowed range. The shell enables the consistent matching and the traversability, but the causal boundary remains. Any attempt to recover the lost history with a second wormhole must therefore either create a new causal path that bypasses the original past horizon or generate closed timelike curves that connect post-crossing events back to the filtered region. The filtering role of the past horizon thus sets a fundamental limit on what additional wormholes can achieve without fundamentally altering the causal structure already fixed by the first bridge.

7. Why a Second Identical Wormhole Cannot Restore Lost History

A second wormhole constructed identically to the first cannot restore the causal history that is lost upon crossing the original throat. The limitation follows directly from the same horizon identification and causal filtering that make the first bridge traversable. Because the second bridge must respect the identical causal structure, it inherits the same past horizon and therefore cannot reach the portion of the past that was already filtered by the first crossing.

Suppose an observer has crossed from Region I into Region II through the original throat and has thereby lost access to events in Region I that lie behind the past horizon. A second wormhole placed between the same two regions creates another identified throat. Any causal curve that uses this second throat must still approach it from the allowed side of the past horizon on the departure sheet. Geodesics originating from the filtered region have no continuation across either throat. The second bridge therefore provides an alternative route between the two sheets but does not open a route into the already inaccessible wedge of the original region's past.

In the Kruskal-Penrose diagram the second bridge appears as another pair of identified null lines connecting the two diamonds. These new lines must be placed consistently with the existing horizon identification. They can connect points that lie to the future of the original past horizon, allowing additional forward passages or even closed loops between the sheets. However, they cannot intersect the wedge below the past horizon line without violating the causal boundary already fixed by the first identification. The diagram shows that the filtered region remains disconnected from all future events in both diamonds once the original crossing has occurred.

The lightlike character of the thin shell reinforces this limitation. Each shell is localized at its own throat and matched independently to the vacuum geometry. The junction conditions that allow traversability for one shell apply equally to the second, but they do not alter the global causal

boundaries defined by the horizons. Adding a second shell does not remove or relocate the past horizon of the first bridge. Consequently the causal cut experienced by the observer after the first crossing remains in place even if a second crossing is available.

An identical second wormhole can at best provide redundancy or alternative paths for observers who have not yet crossed the first bridge. For an observer who has already crossed and lost part of the original past, the second bridge offers no new causal access. Any signal or worldline that attempts to reach the lost events would still have to originate from behind the past horizon relative to the first crossing event. The second throat cannot circumvent this geometric constraint without changing the horizon identification itself.

The same conclusion holds if the second wormhole connects the two regions at a different radial distance or at a different Kruskal time. As long as the identification of future and past horizons is preserved on each shell, the causal filtering associated with the past horizon of the first crossing remains operative. The observer who has already crossed cannot use the second bridge to send a signal backward in time relative to the original crossing in order to query the filtered events. Such a maneuver would require a closed timelike curve that loops around the filtering surface, which an identical second bridge does not automatically provide.

In short, duplication of the original construction merely replicates the same causal structure rather than repairing it. The loss of past history is a consequence of the horizon identification required for traversability in this model. A second identical wormhole inherits that identification and therefore cannot reach behind the original past horizon. Recovery of the lost history requires configurations that go beyond simple replication, either by generating closed timelike curves or by introducing horizon identifications that differ qualitatively from those of the first bridge. The causal structure fixed by the Kruskal-Penrose diagram shows why an identical second wormhole is insufficient.

8. Geometric Requirements for Recovering Causal Past with Additional Wormholes

Recovering the causal history lost upon crossing the original throat requires additional wormholes whose geometry differs from simple duplication of the first bridge. The second wormhole must create causal paths that connect post-crossing events back to the filtered region behind the past horizon of the first crossing. This cannot be achieved by placing an identical lightlike thin-shell bridge at another location between the same two sheets. Instead, the additional structure must either generate closed timelike curves or introduce horizon identifications that allow access to the previously inaccessible wedge.

In the Kruskal-Penrose diagram the lost history occupies the region below the past horizon line in the departure diamond. Any new null surface introduced by a second wormhole must

intersect this region in a way that connects it to future events after the original crossing. Because the original identification already fixes the causal boundary, the new surface must be arranged so that its own horizon identification effectively bypasses or wraps around the filtering surface. One geometric possibility is to place the second throat so that it links a point in Region II (after the first crossing) to a point in Region I that lies on the allowed side of the past horizon but is connected through a longer causal path that loops around the original identification.

A second requirement concerns the relative motion or orientation of the additional throat. If the second wormhole is static and identified in the same manner as the first, its causal structure remains parallel to the original filtering. To create a path that reaches behind the past horizon, the second throat must be positioned or moved in such a way that its future and past horizons are identified with a relative shift in Kruskal time. This shift can allow a future-directed curve in the post-crossing region to connect, after traversing the second throat, to an event that was previously behind the filtering horizon relative to the first crossing. Such a configuration effectively creates a closed timelike curve that loops from the new sheet back into the filtered portion of the original sheet.

The lightlike character of the thin shells imposes further geometric constraints. Each shell must be matched consistently to the vacuum regions, and the identification of horizons on the second shell must not conflict with the identification already present on the first shell. If the two shells intersect or share segments of their null surfaces, the junction conditions must be satisfied simultaneously on the overlapping portions. This consistency requirement limits the possible relative placements of the two throats. In particular, the second shell cannot simply cross the filtered wedge arbitrarily; its null generators must respect the existing causal boundaries while still providing a new connection.

Another geometric possibility involves connecting one of the exterior regions to itself through the second wormhole, forming a handle that links different parts of the same sheet. In the diagram this appears as a null surface that begins and ends on the same diamond. If the identification on this self-connecting throat is chosen so that it links a post-crossing event in Region II back to a pre-crossing event in Region I that lies behind the original past horizon, a closed causal loop can be formed. Such a loop would in principle allow an observer to return to the filtered region after the original crossing. However, the construction of a consistent lightlike thin shell on a self-connecting throat requires careful matching of the null generators and may introduce additional stability conditions on the shell stress tensor.

The relative timing of the two crossings is also constrained. For the lost history to be recoverable, the second wormhole must be traversable at a Kruskal time that allows a future-directed curve from the post-first-crossing region to reach the filtered events before those events fall too far into the past relative to the global structure. If the second throat is placed too

far in the future, the filtered events may already lie beyond any possible causal connection even with the new identification. The diagram shows that the available causal wedges shrink with increasing Kruskal time, so the second bridge must be positioned sufficiently early to intersect the relevant portion of the filtered region.

These geometric requirements show that recovery of lost causal history cannot be achieved by adding another copy of the original bridge. The additional wormhole must introduce either a relative time shift in the horizon identification or a topological handle that links the filtered region to the post-crossing future. Both possibilities go beyond the minimal lightlike thin-shell construction used for the first bridge. They also raise new questions about the stability of multiple interacting null shells and the possible formation of closed timelike curves, which must be examined separately. The diagram of the single-bridge spacetime makes the necessary modifications visible as changes in the placement and identification of the null surfaces that connect the two diamonds.

9. Closed Timelike Curves and Chronology Protection in Multi-Wormhole Configurations

Closed timelike curves offer one geometric route by which lost causal history might be recovered in a multi-wormhole spacetime. In the lightlike thin-shell Einstein-Rosen bridge, the loss of past history occurs because the past horizon filters events behind it from any observer who has crossed the throat. A closed timelike curve that loops from the post-crossing region back into the filtered wedge would in principle allow the observer to return to events that were previously inaccessible. Such a curve requires at least two wormholes whose combined horizon identifications create a topological loop in the causal structure.

In the Kruskal-Penrose diagram a closed timelike curve appears as a worldline that returns to an earlier Kruskal time after traversing multiple identified null surfaces. One possible construction places the second throat so that its future horizon on the arrival sheet is identified with the past horizon on the departure sheet in a shifted manner. A future-directed timelike curve that crosses the first throat, travels in the new region, and then crosses the second throat can emerge at a Kruskal time earlier than the original crossing relative to the filtered events. The net effect is a loop that connects the post-crossing future back to the pre-crossing past that was previously behind the filtering horizon.

The formation of such loops depends on the relative placement and identification of the two null shells. If both shells are static and identified in the same orientation, the causal structure remains acyclic on each sheet. A relative time shift or a spatial offset in the identification of the second shell is required to produce the necessary tilt in the null generators. In terms of the Kruskal null coordinates, this shift corresponds to a nonzero constant added to the identification

map between the vanishing coordinate on one horizon and the non-vanishing coordinate on the other. The resulting geometry contains closed timelike curves provided the shift is large enough relative to the separation between the two throats.

However, the existence of closed timelike curves immediately raises the question of chronology protection. In spacetimes containing such curves, vacuum polarization effects are expected to diverge on the chronology horizon that bounds the region of closed timelike curves. These divergences produce an infinite stress-energy tensor that back-reacts on the geometry and typically destroys the traversability of the wormholes or renders the closed curves unstable. In the lightlike thin-shell construction the shells themselves are already delicate, being supported by exotic matter localized on null surfaces. The additional stress-energy generated by vacuum polarization near a chronology horizon would likely destabilize one or both shells, preventing the consistent maintenance of the horizon identifications required for the closed curves to exist.

Even if the classical geometry admits closed timelike curves, the quantum instability tends to prevent their use for recovering lost history in a controlled manner. An observer attempting to traverse the loop would encounter increasingly strong quantum fluctuations as the chronology horizon is approached. These fluctuations would disrupt both the information carried by the observer and the integrity of the thin shells themselves. Consequently, while closed timelike curves provide a geometric mechanism that could in principle reconnect the observer with the filtered past, the chronology protection mechanism suggests that such curves cannot be stably realized or utilized within the lightlike thin-shell framework.

The interaction between multiple lightlike shells further complicates the situation. When two null shells are present, their mutual gravitational influence and the requirement that each shell remain null impose additional constraints on the possible relative motions and identifications. A configuration that produces closed timelike curves must satisfy the junction conditions on both shells simultaneously while keeping the shells from intersecting in a way that would violate the null character of either surface. These constraints reduce the range of geometries in which stable closed timelike curves could exist. In many cases the only consistent solutions either eliminate the closed curves or force one of the shells to collapse or disperse.

Thus, while closed timelike curves appear necessary for any recovery of lost causal history in this model, their formation in multi-wormhole configurations is both geometrically constrained and likely rendered unstable by quantum effects. The lightlike thin-shell Einstein-Rosen bridge therefore presents a setting in which the causal price of traversability (the filtering by the past horizon) cannot be easily reversed by the addition of further bridges without encountering fundamental obstructions from both classical geometry and chronology protection. Any viable recovery mechanism would have to circumvent or suppress these obstructions while preserving the traversability of each individual throat.

10. Stability of Lightlike Thin Shells Under Multiple-Bridge Perturbations

The lightlike thin shells that support traversability in the Einstein-Rosen bridge are delicate structures whose stability is easily compromised by the introduction of additional wormholes. Each shell is supported by a localized surface stress-energy tensor that must remain null and satisfy the junction conditions with the vacuum geometry on both sides. When a second bridge is added, the gravitational influence and the modified causal structure exert perturbations on the original shell. These perturbations can cause the shell to deviate from null character, disperse, or collapse, thereby destroying the traversability it was intended to maintain.

Classically, the stability of a single lightlike thin shell depends on the balance between the surface energy density and the extrinsic curvature jump across the null hypersurface. In the Kruskal coordinates the shell is placed on a surface of constant radial function at the throat. Small perturbations in the identification map or in the relative position of a second shell translate into variations in the surface stress tensor. Because the shell is null, these variations propagate along the null generators without dispersion. A perturbation that increases the surface energy density beyond a critical threshold can cause the generators to focus and the shell to collapse into a singularity. Conversely, a perturbation that reduces the energy density can cause the throat to reopen uncontrollably or disperse the shell into the surrounding vacuum.

When two or more lightlike shells are present, their mutual gravitational interaction introduces additional degrees of freedom. The null generators of one shell can intersect or graze those of another. At intersection points the junction conditions must be satisfied simultaneously for both shells, which generally requires fine-tuning of the surface stress tensors. Without such tuning, the intersection produces a discontinuity that propagates along the generators and destabilizes at least one of the shells. In the Kruskal-Penrose diagram these intersections appear as crossings of the identified null lines. Each crossing represents a potential site of instability unless the relative strengths of the two shells are precisely matched.

The presence of closed timelike curves, discussed in the previous section, further threatens shell stability. Vacuum polarization near a chronology horizon generates a divergent stress-energy tensor that back-reacts on the geometry. Because the thin shells are already supported by exotic matter, this additional quantum stress-energy tends to violate the null energy condition locally and causes the shells to disperse or collapse. The effect is particularly severe when the chronology horizon intersects one of the shells, as the divergent energy density directly perturbs the surface stress tensor that maintains the throat.

Even in the absence of closed timelike curves, the gravitational influence of a second bridge modifies the effective potential felt by the original shell. In Kruskal coordinates the second

identification shifts the location at which the radial function achieves its minimum on the first shell. This shift acts as a perturbation that can cause the throat to oscillate or drift in Kruskal time. Because the shell is null, such oscillations do not dampen and can grow until the identification between the future and past horizons is lost. Once the identification fails, the traversability of the original bridge is destroyed even if the second bridge remains intact.

The stability problem is compounded by the requirement that both shells remain lightlike. A small timelike or spacelike perturbation introduced by the second bridge propagates along the generators of the first shell and converts the originally null surface into a timelike or spacelike hypersurface over a finite interval. On a timelike or spacelike surface the junction conditions change character, and the exotic matter needed to keep the throat open must satisfy different inequalities. The original lightlike shell cannot accommodate these changed conditions without dispersing or forming caustics.

These stability considerations impose strong restrictions on any attempt to recover lost causal history with additional wormholes. Configurations that produce the closed timelike curves or horizon shifts needed to reach the filtered past almost inevitably perturb the original shell beyond its stability threshold. The resulting instability either collapses the first throat, rendering the original crossing impossible, or disperses the exotic matter, eliminating the traversability of both bridges. Consequently, even if a geometric arrangement exists in principle that could reconnect an observer with the lost portion of the past, the dynamical instability of the lightlike thin shells under multiple-bridge perturbations renders such an arrangement unrealizable in practice.

The interplay between classical stability and quantum chronology protection therefore creates a double barrier. Classically, the shells cannot tolerate the perturbations required for history recovery. Quantum mechanically, any configuration that overcomes the classical barrier tends to generate divergences that further destabilize the shells. Within the lightlike thin-shell framework, these combined effects suggest that lost causal history cannot be stably recovered by the addition of further bridges without fundamentally altering the nature of the supporting shells or abandoning the horizon identification that defines the original construction.

11. Implications for Information and Observers in Traversable Wormhole Spacetimes

The irreversible loss of causal history upon crossing the throat has direct consequences for the information that observers and signals can carry between the two sheets. In the lightlike thin-shell Einstein-Rosen bridge an observer who traverses from one exterior region into the other retains only the portion of the original past that lies to the future of the past horizon at the moment of crossing. All events whose connecting null geodesics originate behind that horizon

are permanently excluded from the traveler's new causal past. This truncation means that the observer arrives in the destination region with an incomplete record of the departure region's history.

For a human or robotic observer the practical effect is a selective amnesia regarding the filtered events. Memories, recordings, or data that were formed from signals originating behind the past horizon are absent after the crossing. The observer can still possess information acquired before the crossing from events that were causally connected through the allowed side of the horizon, but any information that required signals from the filtered wedge is missing. Because the loss is geometric rather than psychological, no amount of internal processing or data compression can recover what was never received. The observer therefore enters the new sheet with a causally incomplete description of the original sheet's past.

Signals and transmitted data suffer the same restriction. A message or data packet sent through the throat carries only the information that was contained in its past light cone up to the transmission event. If the message is emitted after an observer has already crossed, it cannot include information from the filtered region of the departure sheet. Conversely, a message sent from the departure sheet before the crossing can reach the destination only if its null geodesics lie on the allowed side of the past horizon. Messages originating from behind the horizon never arrive. The throat therefore functions as a causal filter for information as well as for worldlines.

This filtering has consequences for any attempt to use the wormhole for scientific observation or communication between the two sheets. An observer who crosses and then attempts to send a report back through a second wormhole can transmit only information acquired after the original crossing or from the allowed portion of the original past. Any data concerning events behind the past horizon of the first crossing cannot be included. The second wormhole, even if it exists, cannot retrieve information that was already lost at the first crossing. The net result is that the two sheets can exchange information forward in time across the throats, but they cannot exchange complete historical records of each other's pasts.

The situation differs markedly from horizon-free traversable wormholes. In those constructions an observer can in principle maintain continuous causal contact with events on both sides before and after crossing, allowing the transfer of complete historical data sets. The retained past horizon in the lightlike thin-shell construction prevents such complete transfer. The observer who crosses must accept that part of the departure region's history will never be accessible again, either directly or through subsequent wormholes, unless the additional structures discussed in earlier sections are introduced. Even then, the stability and chronology-protection issues make reliable recovery of the missing information problematic.

For macroscopic observers the loss of causal history also raises questions about identity and continuity of experience. An observer who crosses experiences a sudden and permanent change in the set of events that can influence their future. While the observer's internal state at the moment of crossing is continuous, the set of possible future influences is abruptly reduced. This reduction is not felt as a psychological shock but as an objective limitation on what can ever be known or verified about the original region after the crossing. In any protocol that requires verification of past events—such as scientific observation, legal records, or historical documentation—the crossing observer must accept that certain facts from the departure sheet can never be confirmed from the destination sheet.

The implications extend to the global consistency of physical laws across the two sheets. Because each sheet retains its own complete set of local physical laws, the loss of causal history does not create contradictions within either sheet individually. However, it does mean that the two sheets cannot serve as complete mutual backups for historical information. Any attempt to use one sheet as a repository for the complete history of the other is frustrated by the filtering action of the past horizon. The wormhole allows spatial connection and forward information flow but enforces a permanent informational asymmetry between the pre-crossing and post-crossing states of any traveler.

In summary, the causal structure of the lightlike thin-shell Einstein-Rosen bridge imposes a fundamental limitation on the information that can be carried or recovered across the throat. Observers lose access to part of the original past upon crossing, and this loss cannot be repaired by simple duplication of the bridge. The filtering action of the past horizon therefore represents not only a loss of causal history but also a loss of informational completeness that persists even in the presence of additional wormholes, unless those wormholes are arranged in configurations that overcome both the classical stability constraints and the chronology-protection effects discussed previously.

12. Conclusions and Open Questions

The lightlike thin-shell reinterpretation of the original Einstein-Rosen bridge yields a traversable connection between two asymptotically flat exterior regions while retaining the causal filtering action of the past horizon. In the associated Kruskal-Penrose diagram the two exterior regions are joined at an identified throat. Future-directed causal curves can cross from one region into the other, yet any observer who crosses permanently loses access to the portion of the original region's causal past that lies behind the past horizon. This loss is geometric and irreversible within the single-bridge spacetime: the identification that enables traversability simultaneously

defines a causal boundary that prevents filtered events from reaching the traveler after the crossing.

The addition of further wormholes does not automatically restore the lost history. A second bridge constructed identically to the first inherits the same horizon identification and therefore cannot reach events behind the original past horizon. Recovery requires configurations that either generate closed timelike curves capable of looping back into the filtered region or introduce horizon identifications with relative time shifts that bypass the filtering surface. Both possibilities demand precise geometric arrangements of multiple null shells and raise new consistency conditions on the surface stress tensors.

Even when such arrangements exist classically, two major obstacles remain. First, the lightlike thin shells are sensitive to perturbations. The gravitational influence and modified causal structure introduced by additional bridges tend to destabilize the original shell, causing it either to collapse or to disperse. Second, any configuration that produces closed timelike curves is expected to encounter chronology protection. Vacuum polarization near the chronology horizon generates divergences that back-react on the geometry and further destabilize the shells. These combined classical and quantum effects create a double barrier against stable recovery of lost causal history.

The analysis therefore shows that the causal price of traversability in this model is a permanent truncation of the observer's past light cone upon crossing. Information and worldlines can move forward from one sheet to the other, but complete historical records cannot be transferred or recovered without overcoming stability and chronology-protection constraints that appear fundamental within the lightlike thin-shell framework.

Several open questions remain. It is not yet clear whether there exist classically stable multi-shell configurations that produce closed timelike curves without triggering immediate collapse of at least one throat. The precise back-reaction of vacuum polarization on null shells near chronology horizons requires detailed calculation within the thin-shell junction formalism. It is also unknown whether quantum effects could be arranged to suppress chronology protection sufficiently to allow controlled recovery of lost history, or whether such effects inevitably destroy the traversability that the shells were introduced to provide. Finally, the extension of the present analysis to charged or rotating generalizations of the lightlike thin-shell bridge remains to be carried out; additional horizons and ergoregions may alter both the filtering mechanism and the stability of multiple-bridge configurations.

These questions indicate that while the lightlike thin-shell Einstein-Rosen bridge provides a controlled arena in which traversability and irreversible loss of causal history coexist, the recovery of lost history through additional wormholes faces substantial geometric, dynamical,

and quantum obstacles. The Kruskal-Penrose diagram makes the origin of the loss and the barriers to its reversal geometrically transparent, but the full resolution of whether and how such recovery can be achieved lies beyond the scope of the single-bridge and minimal multi-bridge constructions examined here.

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